# Modal Analysis of Open Groove Guide with Arbitrary Groove Profile

Zhewang Ma, Eikichi Yamashita, *Fellow, IEEE*, and Shanjia Xu, *Senior Member, IEEE*

***Abstract*— A modal analysis method is proposed for groove guide with arbitrary groove profile. The formulation is versatile, efficient, and rigorous except the step approximation process of the groove profile. Numerical results have shown excellent agreement with previous data.**

## I. Introduction

Groove guide was proposed more than 20 years ago and has been considered promising as a low-loss waveguide for use at millimeter wavelength [1]–[5]. More recently, a theoretical and experimental analysis of a single V-groove guide has been presented [4], which indicated that compared with the rectangular-groove guide, the attenuation of the V-groove guide is low and its rejection capability of higher order modes is effective. The analytical technique employed in [4] was the conformal mapping whose formulation was much complicated and tedious. For arbitrarily shaped groove guides, this method is hardly applicable.

The flexibility of the finite-element method (FEM) and the finite-difference method (FDM) may be used to deal with groove guides with arbitrary cross-sections. But a well-known shortcoming of these methods is the requirement of large computer memory and time-consuming computations. Besides, in order to deal with the infinitely extended two open ends of the groove guide, it is usually necessary to assume that perfect conductor planes are placed on the two sides of the guide far away from the groove in order to form a closed structure. Such a pre-assumption is also necessary for the conventionally used transverse resonance technique in analyzing the groove guides [5] and the groove guide couplers [6].

In this letter, for the first time, a modal analysis method is presented for arbitrarily profiled groove guides. This theory takes into account the effect of higher order modes at all the step discontinuities in a concise and systematic way, thus is rigorous except the step approximation process. Moreover, the two open ends are represented by two *zero matrices* whose elements are zero, so that the above stated preassumption is avoided. The final eigenvalue matrix is simple and no matter how many steps there are, the size of it maintains small and unchanged.

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Z. Ma and E. Yamashita are with the University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182, Japan.

S. Xu is with the University of Science and Technology of China, Hefei, Anhui, 230026, P.R.C.

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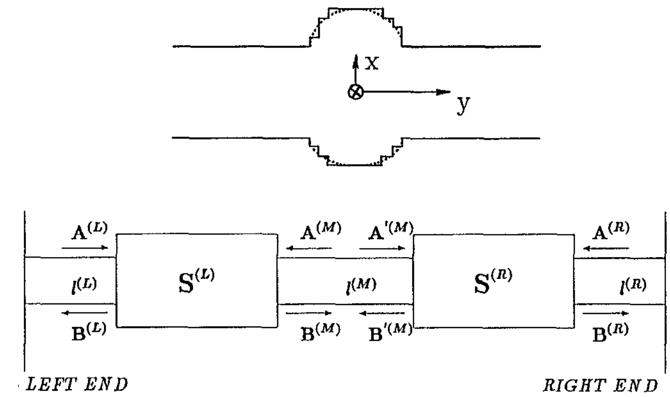


Fig. 1. (a) An arbitrarily profiled groove guide and its step approximation. (b) Application of transverse resonance conditions.

## II. Theory

Fig. 1(a) shows the cross section of an arbitrarily profiled groove guide and its step approximation. The hybrid fields in the guide may be expressed as a superposition of the LSE and LSM modes with respect to the $z$-direction (i.e., the TE and TM modes with respect to the $y$-direction) [3], [7]. We consider that the LSE and LSM modes propagate in the transverse direction and couple each other at discontinuities of various vertical planes. The hybrid modes as waveguide fields are formed as a result of repeated reflections of the LSE and LSM mode waves at the left and right ends and the step discontinuities. From this point of view, the scattering matrices of individual step junctions are derived by using the mode-matching method, and the overall scattering matrix of the cascaded discontinuities is obtained by using the well-known generalized scattering matrix technique [8].

Out of uniform sections forming the series of step discontinuities, we choose an arbitrary one, and use $S^{(L)}$ and $S^{(R)}$ to indicate the overall scattering matrices of the cascaded discontinuities on its left and right side, respectively, as indicated in Fig. 1(b). The forward and backward wave amplitude column vectors in the left, internal, and right sections are related by, respectively,

$$A^{(L)} = D^{(L)} D^{(L)} B^{(L)}, \qquad A^{(R)} = D^{(R)} D^{(R)} B^{(R)}, \quad (1)$$

$$A^{(M)} = D^{(M)} B'^{(M)}, \qquad A'^{(M)} = D^{(M)} B^{(M)}, \quad (2)$$

where $D^{(L)}, D^{(R)}$ and $D^{(M)}$ are diagonal matrices with diagonal elements $D_{nn} = e^{-jk_{yin}l}$. Substituting (1) and (2)

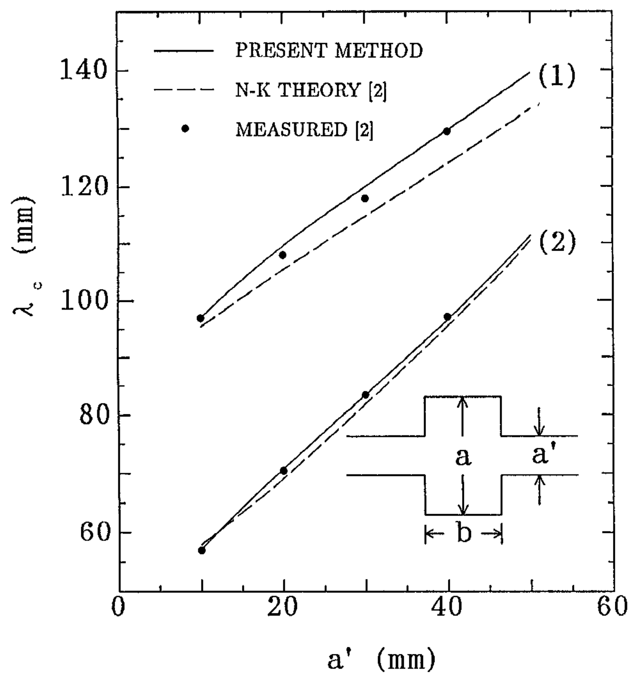


Fig. 2. Comparison between measured and theoretical values of the cutoff wavelength $\lambda_c$ for groove guides of various cross sections; (1) $a = a' + 40$ mm, $b = 20$ mm, (2) $a = a' + 20$ mm, $b = 20$ mm.

into the scattering matrix expressions of $S^{(L)}$ and $S^{(R)}$, the amplitude column vectors $A^{(L)}$, $B^{(L)}$, $A^{(R)}$, $B^{(R)}$, $A^{(M)}$, and $A'^{(M)}$, may be eliminated and a linear simultaneous equations of $B^{(M)}$ and $B'^{(M)}$ are obtained. Its determinant should vanish for the existence of nontrivial solutions, so that the eigenvalue equation is derived. In the case of a groove guide, the left and right end extend to infinity, and the diagonal matrices, $D^{(L)}$ and $D^{(R)}$, representing wave reflections at the two end boundaries become *zero matrices* whose matrix elements are zero, so that the final eigenvalue equation is much simplified as

$$\text{Det}\, G = 0, \qquad G = I - S_{22}^{(L)} D^{(M)} S_{22}^{(R)} D^{(M)}. \tag{3}$$

This process is similar to that of a recent paper [9] for the analysis of MMIC transmission line characteristics, but one main different point is that, in our treatment of the cascaded discontinuities, all the amplitude coefficients are normalized so that the elements of the final scattering matrices, $S^{(L)}$ and $S^{(R)}$, are of order 1, and the eigenvalue matrix $G$ is diagonal dominant. This property makes the numerical calculation process quite stable and greatly eases the root searching process for the eigenvalues.

## III. Numerical Results

In Fig. 2, the cutoff wavelengths for rectangular-groove guides of two dimensions are provided, and a better agreement has been shown between our theory and the carefully measured data than that of the first-order approximation theory of Nakahara and Kurauchi [4].

Table I shows the comparison between the theoretical and experimental guide wavelengths of various V-groove guides at 100 GHz. We found that the correlation is quite good with errors of about 0.1%.

TABLE I
Guide Wavelength $\lambda_0 = 3.117$ mm ($b = a - a'$)

| Guide Dimensions | | Theoretical Calculations | Experimental Results [4] | % of Error |
|---|---|---|---|---|
| $a'$ (mm) | $a$ (mm) | $\lambda_g$ (mm) | $\lambda_g$ (mm) | |
| 10 | 14 | 3.153 | 3.156 | 0.09 |
| 10 | 18 | 3.142 | 3.146 | 0.13 |
| 10 | 22 | 3.133 | 3.136 | 0.10 |
| 12 | 16 | 3.143 | 3.146 | 0.10 |
| 12 | 20 | 3.137 | 3.138 | 0.03 |
| 12 | 24 | 3.131 | 3.130 | 0.03 |

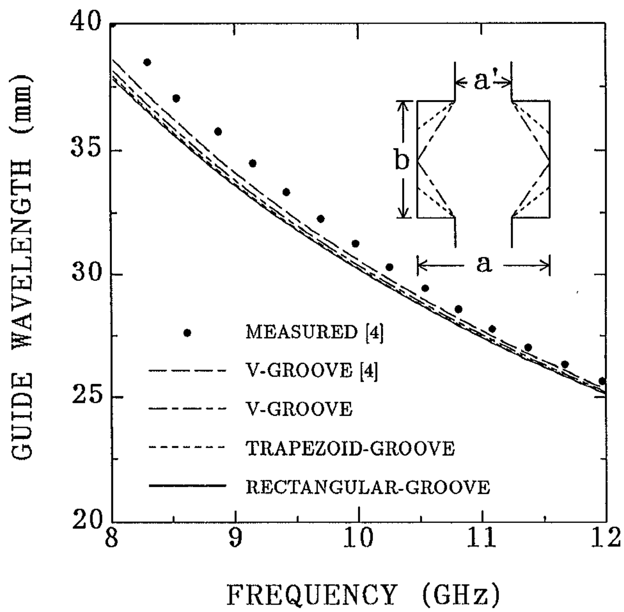


Fig. 3. Guide wavelengths versus frequency in X-band for various groove profiles; $a$ = 165 mm, $a'$ = 75 mm, $b$ = 45 mm.

Finally, dispersion curves for various groove profiles are compared in Fig. 3. Experimental and calculated results of [4] are also plotted for the V-groove guide. Since a movable short-circuited plunger was used at 100 GHz, errors duo to the coupling holes of the end plates were removed. Therefore, the discrepancy between the predicted and measured guide wavelength is significantly higher in X-band than that at 100 GHz [4].

On the other hand, we can see that the effect of the groove profile on the dispersion characteristics is very small. It means that during the fabrication of desired groove guides the demand for dimensional tolerances is not high. This is particularly advantageous over other transmission lines at millimeter wavelength.

And also because of this, a rough step approximation for an arbitrarily shaped grooved guide may yield results with high accuracy. For the V-grooved guide in our computation, we find that the error between the result for a 5-step division and that for a 10-step division is within 1%.

For the correct convergence of numerical results, modes in every subregions are retained according to the ratios between the subregion heights. This is a commonly used technique in the mode-matching method. Fast convergence is found by the present technique, and 3 LSE and 3 LSM modes in the two end guides, respectively, are adequate for obtaining converging results.

## IV. Conclusion

A versatile and efficient modal analysis method for arbitrarily profiled groove guides has been proposed and checked by numerical examples. Further applications of this approach to double-grooved guides, dielectric loaded groove guides, and to the design of broad-band groove guide coupler is undergoing.

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